Set No - 1

I B. Tech I Semester Regular Examinations Feb./Mar. - 2014 **MATHEMATICS-I**

(Common to All Branches)

Time: 3 hours

Max. Marks: 70

Question Paper Consists of **Part-A** and **Part-B** Answering the question in **Part-A** is Compulsory, Three Questions should be answered from **Part-B**

PART-A

- Find the orthogonal trajectories of the curve $r = a(1 + \cos \theta)$. 1.(i)
- (ii) If $x = rsin\theta cos\varphi$, $y = rsin\theta sin\varphi$, $z = rcos\theta$, find $\frac{\partial(r,\theta,\varphi)}{\partial(x,y,z)}$, given that $\frac{\partial(x,y,z)}{\partial(r,\theta,\varphi)} = r^2 sin\theta$. (iii) Find the Laplace transform of $f(t) = \begin{cases} t, & 0 < t < 1 \\ 0, & t > 1 \end{cases}$ using Heaviside function. (iv) Let the heat conduction in a thin metallic bar of length L is governed by the equation
- $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$, t > 0. If both ends of the bar are held at constant temperature zero and the bar is initially has temperature f(x), find the temperature u(x,t).
- (v) Solve $p^2 + pq = z^2$.
- (vi) Find $\frac{1}{D^2-4D+4}x^2 sinx$.

[4+4+4+4+3+3]

PART-B

- $y(2x^2 xy + 1)dx + (x y)dy = 0$ 2.(a) Solve
 - (b) Find the complete solution of $y'' + 2y = x^2 e^{3x} + e^x \cos 2x$ [8+8]
- 3.(a) Solve $\frac{dy}{dx} + x\sin 2y = x^3 \cos^2 y$
 - (b) Find the solution of $\frac{d^2y}{dx^2} + 4y = \sin 3x + \cos 2x$. [8+8]
- 4.(a) Find the Laplace transform of $f(t) = \frac{\cos at \cos bt}{t}$.
 - (b) If $x = \sqrt{vw}$, $y = \sqrt{uw}$, $z = \sqrt{uv}$ and $u = r \sin \theta \cos \varphi$, $v = r \sin \theta \sin \varphi$ and $w = r \cos \theta$, find $\frac{\partial (x,y,z)}{\partial (r \theta, \varphi)}$ [8+8]
- Expand $f(x, y) = e^y \ln(1 + x)$ in powers of x and y using MacLaurin's Series
- Solve $y'' 8y' + 15y = 9te^{2t}$, y(0) = 5 and y'(0) = 10 using Laplace transforms [8+8]
- 6.(a) Solve $(y + xz)p (x + yz)q = x^2 y^2$.
 - Solve the partial differential equation px+qy = 1. [8+8]
- 7.(a) Find the partial differential equation of all spheres whose centers lie on z-axis.
 - Find the solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$, if the initial deflection is

$$f(x) = \begin{cases} \frac{2k}{l}x & \text{if } 0 < x < l/2\\ \frac{2k}{l}(l-x) & \text{if } \frac{l}{2} < x < l \end{cases}$$
 and initial velocity equal to 0. [8+8]

Set No - 2

I B. Tech I Semester Regular Examinations Feb./Mar. - 2014

MATHEMATICS-I

(Common to All Branches)

Time: 3 hours

Max. Marks: 70

Question Paper Consists of **Part-A** and **Part-B** Answering the question in **Part-A** is Compulsory, Three Questions should be answered from Part-B

PART-A

Find the complete solution of $(D^4 + 16)y = 0$. 1.(i)

(ii) If $x = r\cos\theta$, $y = r\sin\theta$, z = z, find $\frac{\partial(r,\theta,z)}{\partial(x,y,z)}$, given that $\frac{\partial(x,y,z)}{\partial(r,\theta,z)} = r$.

(iii) Solve $x^2p^2 + y^2q^2 = z^2$.

(iv) Find the solution, by Laplace transform method, of the integro-differential equation $y' + 3y + 2 \int_0^t y(t) dt = t$

Find the differential equation of the orthogonal trajectories for the family of parabola through the origin and foci on y-axis.

(vi) Find the solution of wave equation in one dimension using the method of separation of variables.

[3+3+4+4+4+4]

 $y(y^2 - 2x^2)dx + x(2y^2 - x^2)dy = 0$ 2.(a)

Find the complete solution of y'' + 5y' - 6y = sin4x sinx. (b)

[8+8]

 $\cos x \, dv = v(\sin x - v) dx$. 3.(a)

Find the solution of $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 2xe^{3x} + 3e^x \cos 2x$. (b)

[8+8]

Find the Laplace transform of $f(t) = \int_0^t e^{-u} \cos u \, du$. 4.(a)

(b) Find the shortest distance from origin to the surface $xyz^2 = 2$.

[8+8]

Find $\frac{\partial(u,v)}{\partial(r,\theta)}$ if u = 2axy and $v = a(x^2 - y^2)$, where $x = r\cos\theta$ and $y = r\sin\theta$. Solve y'' - 8y' + 15 $y = 9te^{2t}$, y(0) = 5 and y'(0) = 10 using Laplace transforms 5.(a)

(b) [8+8]

6.(a)Form the partial differential equation by eliminating the arbitrary function from xyz = f(x + y + z).

Find the solution of $(D^2 - DD' - 2D'^2)z = (y - 1)e^x$, where $D = \frac{\partial}{\partial x}$ and $D' = \frac{\partial}{\partial y}$. [8+8]

7.(a) Solve the partial differential equation xzp + yzq = xy.

(b) Find the temperature in a bar of length l which is perfectly insulated laterally and whose ends O and A are kept at 0°C, given that the initial temperature at any point P of the rod is given by f(x).

[8+8]

Set No - 3

I B. Tech I Semester Regular Examinations Feb./Mar. - 2014 **MATHEMATICS-I**

(Common to All Branches)

Time: 3 hours

Max. Marks: 70

Question Paper Consists of Part-A and Part-B Answering the question in **Part-A** is Compulsory, Three Questions should be answered from **Part-B**

PART-A

- Find the dimensions of rectangular box of maximum capacity whose surface area is S. 1.(i)
 - Find the orthogonal trajectories of the family of curves $x^{2/3} + y^{2/3} = a^{2/3}$. (ii)
 - (iii) A generator having emf 100 volts is connected in series with a 10 ohm resistor and an inductor of 2 henries. If the switch is closed at a time t = 0, find the current at time t > 0.
 - (iv) Find the Laplace transform of $f(t) = \begin{cases} t, & 0 < t < 1 \\ 0, & t > 1 \end{cases}$ using Heaviside function.
 - Solve pq+qx = y.
 - (vi) Find the solution of $2x \frac{\partial z}{\partial x} 3y \frac{\partial z}{\partial y} = 0$ by the method of separation of variables.

- y(1+xy)dx + x(1-xy)dy = 02.(a)
 - Find the complete solution of $y'' + 4y = e^x \sin^2 x$. (b)

[8+8]

- Solve $2x y' + y = \frac{2x^2}{v^3}$, y(1) = 2. 3.(a)
 - Find the solution of $\frac{d^2y}{dx^2} 4\frac{dy}{dx} 5y = e^{2x} + 3\cos(4x + 3)$. (b)

[8+8]

- Find the Laplace transform of $f(t) = te^{-2t}\cos t$. 4.(a)
 - Find the maxima and minima of $x^3 + 3xy^2 15x^2 15y^2 + 72x$. (b)

[8+8]

- 5.(a)
- Expand $f(x, y) = e^{xy}$ in powers of (x-1) and (y-1). Solve y'' + 7y' + 10 $y = 4e^{-3t}$, y(0) = 0 and y'(0) = -1 using Laplace transforms. (b)

[8+8]

- Form the partial differential equation by eliminating the arbitrary constants 'a' and 'b' 6.(a)from $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$.
 - Find the solution of $(4D^2 + 12DD' + 9D'^2)z = e^{3x-2y}$, where $D = \frac{\partial}{\partial x}$ and $D' = \frac{\partial}{\partial y}$. (b)
- 7.(a)Solve the partial differential equation $p \tan x + q \tan y = \tan z$.
 - (b) A tightly stretched string with fixed end points x=0 and x=1 is initially in a position given by $y = y_0 sin^3 \frac{\pi x}{l}$. If it is released from rest from this position, find the displacement v(x,t).

[8+8]

Set No - 4

I B. Tech I Semester Regular Examinations Feb./Mar. - 2014 **MATHEMATICS-I**

(Common to All Branches)

Time: 3 hours

Max. Marks: 70

Question Paper Consists of **Part-A** and **Part-B** Answering the question in **Part-A** is Compulsory, Three Questions should be answered from Part-B

PART-A

- Find the distance from the centre at which the velocity in simple harmonic motion 1.(i)will be 1/3rd of the maximum.
 - Find a point with in a triangle such that the sum of the squares of its distances from the (ii) three vertices is minimum.
 - (iii) Find the solution, by Laplace transform method, of the integro-differential equation $y' + 4y = \int_0^t y(t)dt$, y(0) = 0.
 - (iv) Uranium disintegrates at a rate proportional to the amount present at that time. If M and N grams of Uranium that rae present at times T₁ and T₂ respectively, find the half life of Uranium.
- Find the complete solution of $(D^3 3D^2D' + 3D'^2 D'^3)z = 0$. (v)
- Solve $z^2 = 1 + p^2 + q^2$. (vi)

6.(a)

[4+4+4+4+3+3]

 $\frac{\mathbf{PART-B}}{(3y^2 + 4xy - x)dx + x(x + 2y)dy} = 0$ 2.(a)

Find the solution of $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 6y = \sin 4x \cos x$. (b)

[8+8]

- Find the complete solution of $y'' + 2y = x^2 e^{3x} + e^x \cos 2x$. 3.(a)
 - Solve $x z' + z \log z = z(\log z)^2$. (b)

[8+8]

- Find the Laplace transform of $f(t) = te^{2t}\cos 2t$. 4.(a)
 - If $u = \sin^{-1}(\frac{x^3 + y^3}{\sqrt{x} + \sqrt{y}})$, prove that $xu_x + yu_y = \frac{5}{2} \tan u$. (b)

[8+8]

- If w = (y z)(z x)(x y), find the value of $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z}$
 - Solve $y'' + 2y' + 5y = e^{-t} \sin t$, y(0) = 0 and y'(0) = 1 using Laplace transforms. (b)

Form the partial differential equation by eliminating the arbitrary constants 'a' and 'b'

- from $z = ax + by + a^2 + b^2$. Using method of separation of variables, solve $u_{xt} = e^{-t} cos x$ with u(x, 0) = u(0, t) = 0. (b)
- [8+8]
- Find the temperature in a thin metal rod of length L, with both ends insulated and with 7.(a)initial temperature in the rod is $sin(\frac{\pi x}{L})$.
 - (b) Solve the partial differential equation $p x^2 + qy^2 = z^2$.

[8+8]